Aversch Johnson Affect

Crew (1979) demonstrated the Aversch-Johnson affect mathematically. Take the non-regulated profit maximizing case but assume the utility has no market power and so takes P as given. If the utility is maximizing profits they must be choosing inputs in order to minimize costs. Lets see how the monopolist should choose inputs to maximize profits and thus minimize costs. Write profits as

$$\Pi = P*Q(K,V) - wV - rK$$

Where: Π = profit

P = price of electricity

Q = output of electricity and is a function of V and K

K = quantity of capital used

V = quantity of variable inputs labor and fuel

w = cost per unit of variable inputs

r = cost per unit of capital

One way to compute the unit cost of capital or sometimes called the user of cost of capital is as in Jorgenson (1975)

$$r = (i + d)*P_{Idef}$$

where i = long term interest rate

d = constant rate of economic depreciation

P_{Idef} = capital investment deflator in year t

The firm should pick capital and variable inputs to maximize profits. Calculus tells us the first order conditions for a maximum are

$$\Pi_K = P*\partial Q/\partial K - r = 0 \;\; => \;\; P*\partial Q/\partial K = r$$

$$\Pi_V = P*\partial Q/\partial V - w = 0 \;\; => P*\partial Q/\partial V = w$$

$\partial Q/\partial K$ is the marginal product of capital or the amount of extra electricity produced by an extra unit of capital. If we multiply the extra output for a unit of capital times the price of the output, P, we get the extra revenue brought in by a unit of capital. We call this whole expression the marginal revenue product of capital which should be set equal to the price of capital.

Second order conditions require that

$$\Pi_{KK} < 0, \Pi_{VV} < 0 \text{ and } \Pi_{KK}\Pi_{VV} - \Pi^2_{KV} > 0$$

The first two conditions above are the more interesting. Writing these two conditions out for the above formula we get that

$$\Pi_{KK} = P^{*}\partial^2 Q/\partial K^2 < 0$$

$$\Pi_{VV} = P^{*}\partial^2 Q/\partial V^2 < 0$$

For the above two expressions to be negative $\partial^2 Q/\partial K^2$ and $\partial^2 Q/\partial V^2$ have to be negative. Note that these two expressions are the slope of the marginal product curves of capital and variable inputs, respectively. The second order conditions require that these two curves slope down or that we have diminishing marginal products for capital and variable input.

We can rearrange the first order conditions to be

$$\frac{\partial Q/\partial V}{w} = \frac{\partial Q/\partial K}{r}$$

This equation requires that the marginal product of variable input divided by the cost of the variable input equals the marginal product of capital divided by the cost of capital. We should get the same amount of output per dollar of input for each of our factors. If we get more output per dollar of capital, we should buy more capital driving down its marginal product until it equals the marginal product per dollar of variable input.

Another way of writing the first order conditions is

$$\frac{\partial Q/\partial K}{\partial Q/\partial V} = \frac{r}{w} \qquad \text{(1)}$$

The above equation (4.3) indicates that we should operate to make the ratio of marginal products of our inputs equal to the ratio of their prices.

Now we will compare the profit maximizing/cost minimizing situation above with what would happen under rate of return regulation.

Under rate of return regulation the utility is maximizing under the rate of return restraint. We can write this problem as maximizing

$$\Pi = P^{*}Q(K,V) - wV - rK$$

subject to the constraint that

$$\frac{P^{*}Q(K,V) - wV}{K} = s$$

We can rewrite the constraint as

$$P^{*}Q(K,V) - wV = sK$$

Where: s = allowed rate of return

Assume s > r, otherwise there is no incentive to invest since you are just covering your costs of capital. To solve this optimization problem we turn again to calculus and use a Lagrangean function

$$\Im = P^*Q(K,V) - wV - rK - \lambda\,(P^*Q(K,V) - wV - sK)$$

This function looks much like our problem with no constraints except for the extra term that handles the constraint. When the constraint holds the last expression equals zero and $\Im$ is equal to profits. The last expression can be either added or subtracted and the λ in this expression is called the Lagrangean multiplier. We can develop an interpretation of λ by taking the partial of the Lagrangean with respect to s to get

$$\Im_s = \lambda K \Rightarrow \lambda = \Im_s/K$$

Now since $\Im$ equals profits, $\Im_s$ equals the marginal change in profits from a change in regulated rate of return. $\Im_s/K$ equals this change in profit per unit of capital stock. So λ equals the change in profit for one unit of capital from a change in the regulated rate of return. $0<\lambda<1$ since the rate of profit increase can't be greater than the allowed change in profit rate

With a Lagrangean function we take the derivative with respect to the choice variables and the Lagrangean multiplier and set them equal to zero for the first order conditions. Note the third equation, the derivative with respect to the Lagrangean multiplier, forces us to be on the constraint.

$$\Im_V = P^*\partial Q/\partial V - w - \lambda(P^*\partial Q/\partial V - w) = 0$$

$$\Im_K = P^*\partial Q/\partial K - r - \lambda(P^*\partial Q/\partial K - s) = 0$$

$$\Im_\lambda = P^*Q(K,V) - wV - sK = 0$$

We can rewrite the first condition as

$$(1-\lambda)\,P\,\partial Q/\partial V = (1-\lambda)w \tag{2}$$

We can rewrite the second condition as

$$(1-\lambda)\,P\,\partial Q/\partial K = -\lambda\,s + r = r - \lambda r + \lambda r - \lambda s$$

Which can be rearranged as

$$(1-\lambda)\,P\,\partial Q/\partial K = (1-\lambda)\,r - \lambda(s-r)$$

Dividing 2 by 1 gives us

$$\frac{\partial Q/\partial K}{\partial Q/\partial V} = \frac{r}{w} - \frac{\lambda(s-r)}{(1-\lambda)w} \tag{3}$$

Since $0<\lambda<1$, $r>0$, $w>0$, and $s>r$, the last expression in (3) is positive. It is being subtracted from the first expression, which gives us

$$\frac{\partial Q/\partial K}{\partial Q/\partial V} < \frac{r}{w} \qquad (4)$$

Remember from our profit maximizing - cost minimizing condition (1)

$$\frac{\partial Q/\partial K}{\partial Q/\partial V} = \frac{r}{w} \qquad (1)$$

The marginal product of capital is lower in 4 than in 1. With diminishing marginal product this suggests that the capital stock is higher under rate of return legislation than under cost minimization.