Ramsey Pricing to Allocate Fixed Costs in a Natural Monopoly

To see what theory tells us about efficiency, we optimize social welfare subject to a break even constraint as in Crew (1979?). Our Lagrangean becomes

$$\Im = \int P_1(X_1)\, dX_1 + \int P_2(X_2)\, dX_2 - C_f - C(X_1, X_2)$$

$$+ \lambda(X_1P_1(X_1) + X_2P_2(X_2) - C_f - C(X_1, X_2))$$

Our first order conditions are

$$\Im_1 = P_1 - C_1 + \lambda\, (P_1 + X_1\partial P_1/\partial X_1 - C_1) = 0$$

$$\Im_2 = P_1 - C_2 + \lambda\, (P_2 + X_2\partial P_2/\partial X_2 - C_2) = 0$$

$$\Im_\lambda = X_1P_1(X_1) + X_2P_2(X_2) - C_f - C(X_1, X_2) = 0$$

We can rearrange our first two first order conditions to be

$$(1+\lambda)(P_1 - C_1) + \lambda\, (P_1(X_1/P_1)\partial P_1/\partial X_1) = 0$$

$$(1+\lambda)(P_1 - C_2) + \lambda\, (P_2(X_2/P_2)\partial P_2/\partial X_2) = 0$$

Noting the price elasticity (ε) in the last expressions we can rewrite them again as

$$(1+\lambda)(P_1 - C_1) + \lambda\, (P_1/\varepsilon_1) = 0$$

$$(1+\lambda)(P_1 - C_2) + \lambda\, (P_2/\varepsilon_2) = 0$$

The first equation can be further rearranged as

$$\frac{(P_1 - C_1)}{P_1} = \frac{-\lambda}{(1+\lambda)\varepsilon_1}$$

Note that the ε's are negative. If we take the absolute value we make them positive and cancel out the negative on the right hand side.

$$\frac{(P_1 - C_1)}{P_1} = \frac{\lambda}{(1+\lambda)\, |\varepsilon_1|}$$

Similarly for equation 2.

$$\frac{(P_2 - C_2)}{P_2} = \frac{\lambda}{(1+\lambda)\, |\varepsilon_2|}$$

Note that the left hand side of each equation is the markup or price minus marginal cost divided by price. The markup is how much of fixed costs are allocated to each good. This result, referred to as Ramsey pricing, suggests that as a good gets more demand elastic the efficient markup should be lower.