

**38. Correct. The answer is true.** Under Stackelberg, both suppliers have some power in the market, with the leading or more powerful firm having more power and moving first in the market. In the Dominant Firm model, the leader (dominant firm) sets the price and determines the equilibrium quantity. The less powerful supplier then takes that price (price taker) and maximizes on the residual demand.

We can verify this using  $Q_t = 100 - P$ ,  $TC_L = 18Q_L + 4$ ,  $TC_F = 2Q_F^2 + 10Q_F$ , where subscript t = total, L = leader (dominant firm) and F = follower (competitive fringe).

Here is how to solve these models:

### 1) Stackelberg

1. We start with the follower firm profit equation since it is determined by the leading firm:

$$\pi_f = PQ_F - TC_F = [100 - (Q_F + Q_L)]Q_F - 2Q_F^2 - 10Q_F$$

FOC:

$$\partial\pi_f/\partial Q_F = [100 - (Q_F + Q_L)] - Q_F - 4Q_F - 10 = 0$$

$$Q_F^* = (90 - Q_L)/6$$

2. The leader's profit equation:

$$\pi_L = PQ_L - TC_L = [100 - (Q_F + Q_L)]Q_L - 18Q_L - 4$$

FOC:

$$\partial\pi_L/\partial Q_L = [100 - (Q_F + Q_L)] - Q_L - 18 = 0$$

$$100 - Q_F - 2Q_L - 18 = 0$$

Substituting for  $Q_F^*$  from previous step:

$$Q_L = (1/2) * [100 - (1/6)(90 - Q_L) - 18]$$

Solving for  $Q_L^*$  we get  $Q_L^* = 30.92$

$$\text{and } Q_F^* = 9.85$$

$$Q_t^* = Q_F^* + Q_L^* = 40.77$$

$$P^* = 100 - 40.77 = 59.23$$

### 2) Dominant Firm

Let the leading firm from previous Stackelberg model be the dominant firm (DF), and the follower firm be the competitive fringe (CF). Note here that profit maximization is based on MC. The DF will set the price and the CF will act as competitive firm since they are price takers and maximize on residual demand.

1. Starting with CF to determine when they will enter the market:

$$P = MC_{CF}$$

$$100 - Q = 4Q + 10$$

$$5Q = 90$$

$$Q = 18 \text{ and } P = 100 - Q = 82$$

Finding P when  $Q_{CF} = 0$

$$P = 4Q + 10 = 4*(0) + 10 = 10$$

The total demand when  $P = 10$

$$Q = 100 - P = 90$$

So far, we have generated two points for the straight line of residual demand with vertical intercept  $(P, Q) = (82, 0)$  and intersection with total demand curve at  $(10, 90)$ . The slope of this line is:

$$b = (\Delta P)/(\Delta Q) = (10 - 82)/(90 - 0) = -72/90 = -0.8$$

2. Now we use the new inverted total demand equation to find equilibrium price  $P^*$  and  $Q_{DF}^*$  set by the DF.

Since  $P = 82 - 0.8Q$ , the DF will operate using  $MR_{DF} = 82 - 1.6Q$

$$MR_{DF} = MC_{DF}$$

$$82 - 1.6Q_{DF} = 18$$

$$Q_{DF}^* = 40 \text{ \& } P^* = 82 - 0.8Q_{DF} = 82 - 0.8(40) = 50$$

Note again that the CF are price takers, so:

$$P^* = MC_{CF}$$

$$50 = 4Q_{CF}^* + 10$$

$$Q_{CF}^* = (50 - 10)/4 = 10$$

Comparing solutions:

Dominant Firm:

$$Q_{DF}^* = 40$$

$$P^* = 50 \text{ \& } Q_{CF}^* = 10$$

Stackelberg:

$$Q_L^* = 30.92$$

$$P^* = 59.23$$

$$Q_F^* = 9.85$$

***Stackelberg equilibrium is lower in quantities and higher in price than the Dominant Firm model.***