

3. Correct. The answer is false. The basic rule is

$$\frac{U_E}{P_E} = \frac{U_N}{P_N}$$

The first order condition tells us to consume at the point where the marginal utility of energy per dollar of energy products equals the marginal utility of non-energy per dollar of non-energy product.

Now let's develop this rule mathematically. The consumer's goal is to maximize utility $U(E,N)$ subject to their budget constraint $P_E E + P_N N = Y$. To maximize subject to a constraint, we turn to calculus and the Lagrangian multiplier technique. In this technique, we optimize the function with the constraint attached. First convert the constraint into an implicit function by moving all variables to the right hand side, $Y - P_E E + P_N N = 0$. Next introduce a new variable called a Lagrangian multiplier (λ). We will see in a minute the interpretation of this new variable. Multiply this variable times the implicit income constraint and add to the objective function. We call this new function a Lagrangian and write as:

$$\mathfrak{J} = U(E, N) + \lambda(Y - P_E E + P_N N)$$

Now we optimize as if E , N and λ are all choice variables. Before we do so however, let's look at the interpretation of λ . Along the constraint, \mathfrak{J} is utility. Then if we take the partial of \mathfrak{J} with respect to income we get $\partial \mathfrak{J} / \partial Y = \lambda$. Thus, λ represents the marginal utility of income.

Next let take a look at our first order conditions:

$$\mathfrak{J}_E = U_E - \lambda P_E = 0 \quad (16.2)$$

$$\mathfrak{J}_N = U_N - \lambda P_N = 0 \quad (16.3)$$

$$\mathfrak{J}_\lambda = Y - P_E E + P_N N \quad (16.4)$$

Notice that the last expression forces us to be along the constraint. Next solve (16.2) and (16.3) for $\lambda = U_E / P_E$ and $\lambda = U_N / P_N$ and set the right hand side expressions equal to each other we get an expression equivalent to our graphical result above:

$$\frac{U_E}{P_E} = \frac{U_N}{P_N} \quad (16.5)$$

$$\mathfrak{J} = U(E, N) + \lambda(Y - P_E E + P_N N)$$

Second order conditions: Chiang (1984) shows that the above condition is a maximum, when:

$$2P_E P_N U_{EN} - P_E^2 U_{NN} - P_N^2 U_{EE} > 0$$

at the extreme point. He also shows that this condition is equivalent to the indifference curves being convex or bowing in towards the origin as drawn in chapter 16.