

## Self Test

### Oil Refining, Energy Transportation, and Linear Programming

Click on True or False to test your knowledge of the chapter.

1. **True False** Crude oil is a chemical compound. (Contributed by Benoit Gervais)
2. **True False** Catalytic cracking allows refineries to increase the amount of gasoline and diesel produced from a barrel of oil. (Contributed by Benoit Gervais edited by Wisam Assiri)
3. **True False** Lighter crude oil is always preferable. (Contributed by Benoit Gervais)
4. **True False** Crude oil's API gravity is its only important characteristic. (Contributed by Benoit Gervais)
5. **True False** A distillation curve shows whether the whole barrel has been distilled or not. (Contributed by Benoit Gervais)
6. **True False** The U.S. possesses the largest refineries in the world. (Contributed by Benoit Gervais).
7. **True False** Normal butane can be used to increase octane level beyond 100. (Contributed by Benoit Gervais).
8. **True False** Blendco Inc. is a gasoline blender that makes regular (R) and premium gasoline (P). Its recipe for each barrel of regular gasoline is  $\frac{3}{4}$  barrel of grade 1 blending stock (B1) and  $\frac{1}{4}$  barrel of grade 2 blending stock (B2). Its recipe for each barrel of premium gasoline is  $\frac{3}{5}$  barrel of grade 1 blending stock and  $\frac{2}{5}$  barrel of grade 2 blending stock. The production functions for R is

$$R = \min[(\frac{4}{3})B1, (4)B2]$$

(Contributed by Benoit Gervais edited by Wisam Assiri)

9. **True False** Blendco Inc. makes a net income of \$1 for each barrel of regular gasoline (R) it produces and \$1.50 for each barrel of premium gasoline (P) it produces. The firm currently has 150 barrels of grade 1 blending stock (B1) available and 85 barrels of grade 2 blending stock available (B2). As above its recipe for each barrel of regular gasoline is  $\frac{3}{4}$  barrel of grade 1 blending stock (B1) and  $\frac{1}{4}$  gallon of grade 2 blending stock (B2). Its recipe for each barrel of premium gasoline is  $\frac{3}{5}$  barrel of grade 1 blending stock and  $\frac{2}{5}$  barrel of grade 2 blending stock. If Blendco's goal is to maximize profits, its problem can be formulated as the following linear program (LP): (Contributed by Benoit Gervais edited by Wisam Assiri and Pan Feng)

$$\text{Maximize } R + 1.5P$$

Subject to:

$$0.75R + 0.25P \leq 150$$

$$0.40R + 0.60P \leq 85$$

$$B1, B2, R, P \geq 0$$

(Contributed by Benoit Gervais)

10. **True**

**False** Take the above LP from Blendco

Maximize  $R + 1.5P$

Subject to:  $0.75R + 0.60P \leq 150$

$0.25R + 0.40P \leq 85$

$B1, B2, G, P \geq 0$

The optimal profits are when the refiner produces 160 barrels of R and 75 barrels of P.  
(Contributed by Benoit Gervais edited by Wisam Assiri)

**11. True**

**False** Take the LP from Blendco

Maximize  $R + 1.5P$

Subject to:  $0.75R + 0.60P \leq 150$

$0.25R + 0.40P \leq 85$

$B1, B2, G, P \geq 0$

At the optimum  $R = 60$ , and  $P = 175$ , the amounts of B1 and B2 blended to R and P are

	R	P
B1	45	105
B2	15	70

(Contributed by Benoit Gervais)

**12. True False** You want to find the cheapest way to transport gas from two underground storage depots (quantities stored are U1 and U2) to two local distribution companies (quantities demanded are D1 and D2). Let  $X_{ij}$  denote the amount of shipment from source  $i$  to destination  $j$  and  $T_{ij}$  be the cost of transport from depot  $i$  to company  $j$ . The LP formulation of the problem to minimizing the total transportation cost is:

$\text{Min } C = T_{11}X_{11} + T_{12}X_{12} + T_{21}X_{21} + T_{22}X_{22}$

$X_{ij} > 0$

First order conditions for this problem are

$C_{X_{11}} = 0,$

$C_{X_{12}} = 0,$

$C_{X_{21}} = 0,$

$C_{X_{22}} = 0.$

(Contributed by Benoit Gervais)

**13. True False** You want to find the cheapest way to transport gas from two underground storage depots (U1 and U2) to two local distribution companies (D1 and D2). The quantities supplied and demanded at each origin depot and each demand center together with the unit transportation cost are summarized in the following table.

### The Unit Transportation Cost Matrix

	D1	D2	Supply
O1	20	30	200
O2	10	40	100
Demand	150	150	300

Let  $X_{ij}$  denotes the amount of shipment from source  $i$  to destination  $j$ . The LP formulation of the problem minimizing the total transportation cost is:

$$\text{Min } 20X_{11} + 30X_{12} + 10X_{21} + 40X_{22}$$

subject to:

$$X_{11} + X_{12} = 200$$

$$X_{21} + X_{22} = 100$$

$$X_{11} + X_{21} = 150$$

$$X_{12} + X_{22} = 150$$

all  $X_{ij} \geq 0$

(Contributed by Benoit Gervais edited by Pan Feng)

**14. True False** Pipelines are the cheapest way to transport natural gas. (Contributed by Benoit Gervais)

**15. True False** We have economies of scale when a greater quantity of goods or services produced by a same entity decreases the cost per unit. If we have a production function  $Q = L^2$ . We expect that we have economies of scale. (Contributed by Benoit Gervais)

**16. True False** The US has the largest number of tankers registered. (Contributed by Benoit Gervais)

**17. True False** Given that the distance between the Arabian Gulf and New York via the Suez canal is 8,318 statute miles and it takes around 15 hours to transit the canal, 468 hours is the minimum time necessary for a 320,000 dwt ship to transport oil between those two points assuming that the ship travels at 16 Knots. (Contributed by Benoit Gervais)

**18. True False** The Suez canal is one of six important tanker bottlenecks at this time. (Contributed by Benoit Gervais)

**19. True False** The charter rate index Worldscale is based on a given tanker speed and size. Everything else being constant, the larger the value of the index, the smaller the tanker. (Contributed by Benoit Gervais)

**20. True False** It is unlikely that a U.S. ship owner may desire to register its tanker in another country. Tankers are sometimes flown under flags of convenience. This means they are registered in a country where tax rates, operating standards and environmental requirements are more lax. Tankers flown under flags of convenience are more often involved in tanker accidents than other tankers. (Contributed by Benoit Gervais)

21. **True False** North America is the largest importer and exporter of petroleum. (Contributed by Benoit Gervais)

22. **True False** Coal transport is a big business. Seaborne transport of steam coal is about 1/4 that of oil by tonnage and is one of the cheapest means of long distance coal transport. (Contributed by Benoit Gervais)

23. **True False** Given 4000 barrels of straight run gasoline with a motor octane of 61.6, 6000 barrels of reformat with a motor octane of 84.4, 1000 barrels of light hydrocrackate with a motor octane of 73.7, 8000 barrels of cat cracked gasoline with a motor octane of 76.8, 3619.5 barrels of normal butane with a motor octane of 92, we would need 2243 barrels of alkylate with an octane of 95.9 to get a blended motor octane of 80. (Contributed by Greta Goto)

24. **True False** According to the information provided below, the refinery's complexity factor is 9.38.

Process Unit	Complexity Index	Share of crude
Distillation	1.0	100%
Thermal Process	6.0	75%
Catalytic Cracking	6.0	30%
Catalytic Reforming	5.0	10%
Thermal Process	2.5	15%
Coking	6.0	20%

Source: W.L. Nelson. 'The Concept of Refinery Complexity.'

*Oil & Gas Journal*. Sept. 13, 1976.

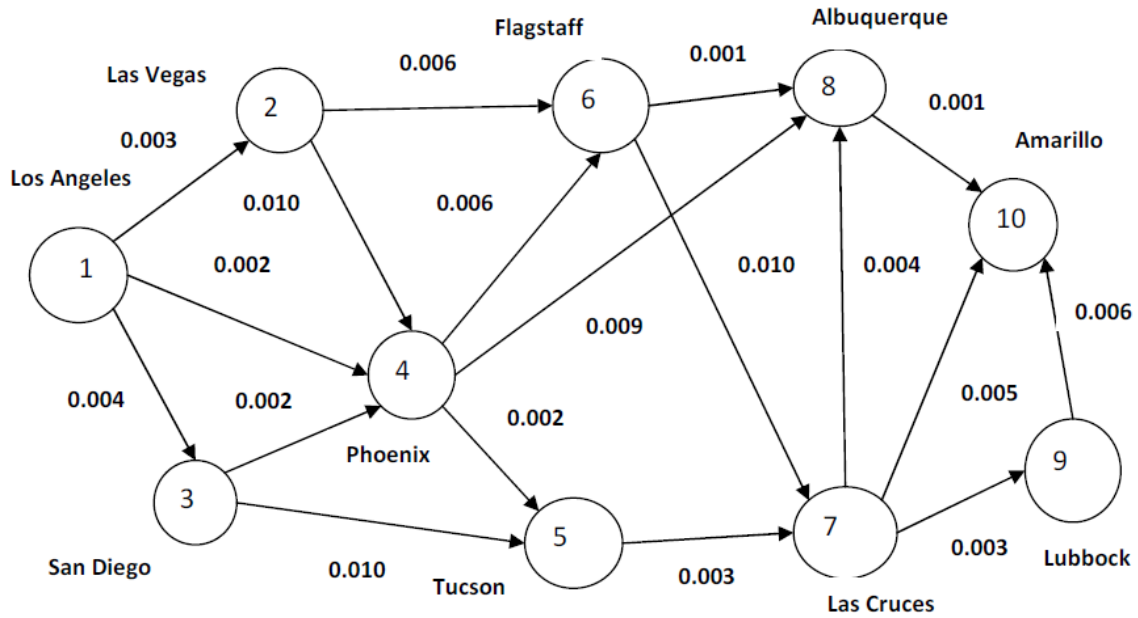
25. **True False**. Linear programming is a powerful tool for solving optimization problems, where the objective function and constraints are all linear.

**For the next more challenging question, you can assume the mathematical model is correct but check to see if you can verify whether the Excel set up is done incorrectly.**

26. **True False**.

Safety Trans is a trucking company that specializes in transporting extremely valuable and extremely hazardous materials. Due to the nature of the business, the company places great importance on maintaining a clean driving safety record. This not only helps keep their reputation up but also keeps their insurance premiums down. The company is also conscious of the fact that when carrying hazardous materials, environmental consequences that even a minor accident could be disastrous.

Safety Trans likes to ensure that it selects routes that are least likely to result in an accident. The company is current currently trying to identify the safest routes for carrying a load of hazardous material from Los Angeles to Amarillo Texas. The following network summarizes the routes under consideration. The numbers on each arc represent the probability of having an accident each potential leg of the journey.



The objective is to find the route that minimizes the probability of having an accident, or equivalently, the route that maximizes the probability of not having an accident.

The model picks the  $x_{ij}$ , which are the binary flows from node  $i$  to node  $j$ .  $x_{ij} = 1$  if the route is taken and 0 if it is not taken.  $P_{ij}$  is the probability of having an accident while traveling from node  $i$  to node  $j$ . Then  $1 - P_{ij}$  is the probability of not having an accident.

Let's use the objective function of maximizing the probability of not having an accident for one unit of travel. The objective function then is to maximize:

$$\begin{aligned}
 & (1 - p_{12}x_{12}) * (1 - p_{13}x_{13}) * (1 - p_{24}x_{24}) * (1 - p_{26}x_{26}) * (1 - p_{34}x_{34}) * (1 - p_{35}x_{35}) * \\
 & (1 - p_{45}x_{45}) * (1 - p_{46}x_{46}) * (1 - p_{48}x_{48}) * (1 - p_{57}x_{57}) * (1 - p_{67}x_{67}) * (1 - p_{68}x_{68}) * \\
 & (1 - p_{78}x_{78}) * (1 - p_{79}x_{79}) * (1 - p_{7,10}x_{7,10}) * (1 - p_{8,10}x_{8,10}) * (1 - p_{9,10}x_{9,10})
 \end{aligned}$$

Next set up the constraints. We require that 1 unit leaves node 1, which we represent as  $-x_{12} - x_{13} - x_{14} = -1$  and 1 unit arrives at node 10, which we represent as  $+x_{7,10} + x_{8,10} + x_{9,10} = 1$ . There must also be a balance at all other nodes with net inflow (inflow - outflow) equal zero. Thus for node 2 the flow in  $x_{12}$  must go to node 4 or node 6 so  $+x_{12} - x_{24} - x_{26} = 0$ . Putting all the nodal constraints together yields:

$$\begin{aligned} \text{Node 1: } & -X_{12} - X_{13} - X_{14} = -1 \\ \text{Node 2: } & +X_{12} - X_{24} - X_{26} = 0 \\ \text{Node 3: } & +X_{13} - X_{34} - X_{35} = 0 \\ \text{Node 4: } & +X_{14} + X_{24} + X_{34} - X_{45} - X_{46} - X_{48} = 0 \\ \text{Node 5: } & +X_{35} + X_{45} - X_{57} = 0 \\ \text{Node 6: } & +X_{26} + X_{46} - X_{67} - X_{68} = 0 \\ \text{Node 7: } & +X_{57} + X_{67} - X_{78} - X_{7,10} = 0 \\ \text{Node 8: } & +X_{48} + X_{68} + X_{78} - X_{8,10} = 0 \\ \text{Node 9: } & +X_{79} - X_{9,10} = 0 \\ \text{Node 10: } & +X_{7,10} + X_{8,10} + X_{9,10} = 1 \end{aligned}$$

We also require that each flow either be 0 or 1, so that each  $x_{ij}$  is binary.

Your assistant has set up the model in excel is as follows:

	A	B	C	D	E	F	G	H	I	J	K	L
1												
2		Select Route1=yes, 2=no, $x_{ij}$	Link	Begin Node	End Node	Probability of an Accident	$(1-P_{ij}*x_{ij})$	Node	Nodal Constraints		Supply=-1, Demand =1	
3	0	12	Los Angeles	Las Vegas	0.003	$=(1-F3*B3)$	1	Los Angeles	$=-B3-B4-B5$	-1		
4	0	13	Los Angeles	San Diego	0.004	$=(1-F4*B4)$	2	Las Vegas	$=B3-B6-B7$	0		
5	0	14	Los Angeles	Phoenix	0.002	$=(1-F5*B5)$	3	San Diego	$=B4-B8-B9$	0		
6	0	24	Las Vegas	Phoenix	0.01	$=(1-F6*B6)$	4	Phoenix	$=B5+B8-B10-B1$	0		
7	0	26	Las Vegas	Flagstaff	0.006	$=(1-F7*B7)$	5	Tucson	$=B9+B10-B13$	0		
8	0	34	San Diego	Phoenix	0.002	$=(1-F8*B8)$	6	Flagstaff	$=B7+B11-B14-B$	0		
9	0	35	San Diego	Tucson	0.01	$=(1-F9*B9)$	7	Las Cruces	$=B13+B14-B16-1$	0		
10	0	45	Phoenix	Tucson	0.002	$=(1-F10*B10)$	8	Albuquerque	$=B12+B15+B16-0$	0		
11	0	46	Phoenix	Flagstaff	0.006	$=(1-F11*B11)$	9	Lubbock	$=B17-B20$	0		
12	0	48	Phoenix	Albuquerque	0.009	$=(1-F12*B12)$	10	Amarillo	$=B18+B19+B20$	1		
13	0	57	Tucson	Las Cruces	0.003	$=(1-F13*B13)$						
14	0	67	Flagstaff	Las Cruces	0.01	$=(1-F14*B14)$						
15	0	68	Flagstaff	Albuquerque	0.001	$=(1-F15*B15)$						
16	0	78	Las Cruces	Albuquerque	0.004	$=(1-F16*B16)$						
17	0	79	Las Cruces	Lubbock	0.003	$=(1-F17*B17)$						
18	0	7,10	Las Cruces	Amarillo	0.005	$=(1-F18*B18)$						
19	0	8,10	Albuquerque	Amarillo	0.001	$=(1-F19*B19)$						
20	0	9,10	Lubbock	Amarillo	0.006	$=(1-F20*B20)$						
21					Probability of a Safe Trip=	$=SUM(G3:G20)$						

In her file, the solution will be in b3:b20. The zeros are starting values. Notice how the objective function is set up in g21 using values in cells c3:g20. G21 could have been one formula but that would be more complicated to set up and more error prone. The nodal constraints are set up in J3:k10. You find your assistant has made one mistake setting up the file.