

**26. Correct. The answer is true.** Chiang (1992), p 253-264 assumes the Solow growth trajectory:

$$\dot{k} = f(k) - c - (\delta + n)k$$

and drives the utility maximizing consumption trajectory.

Chiang picks the optimal consumption/saving path to maximize welfare. Consumption per capita will determine the welfare over time with the social utility function  $U(c)$  with  $U_c > 0$  and  $U_{cc} < 0$  for  $c > 0$  and

$\lim_{c \rightarrow 0} U_c = \infty$  and  $\lim_{c \rightarrow \infty} U_c = 0$ . This function is weighted by population, discounted and summed over all generations to give the objective function as:

$$\int_0^{\infty} U(c)L(t)e^{-\rho t} dt = \int_0^{\infty} U(c)L(0)e^{nt}e^{-\rho t} dt = L(0) \int_0^{\infty} U(c)e^{-(\rho-n)t} dt$$

As above  $n$  is the growth rate of labor;  $\rho$  is the added discount rate. He sets initial labor equal to 1 for convenience, and sets up the following model, which he solves using optimal control:

$$\max_c \int_0^{\infty} U(c)e^{-(\rho-n)t} dt$$

$$\text{subject to } \dot{k} = f(k) - c - (n + \delta)k, k(0) = k_0$$

$$0 \leq c(t) \leq f(k(t))$$

He solves the model for the following consumption per capita growth trajectory.

$$\dot{c} = -\frac{U_c(c)[(f_k - (\delta + \rho))]}{U_{cc}(c)}$$

Since  $U_c > 0$  and  $U_{cc} < 0$ , it shows that if the marginal product of capital per capita is greater than the discount rate plus depreciation, income per capita will be increasing, if it is smaller income per capita will be decreasing. He also shows that a steady state Golden Rule rate is at a lower level than the Golden Rule rate under no discounting.