Pricing a Futures on an Asset Paying Income

The above example was for a financial asset with no income. However, most financial assets such as stock and bonds pay income over their lifetime. Pricing a futures for such an asset with income is only slightly more complicated. All you have to do is first convert the interest or dividend payments into the equivalent loan taken out today at the risk free rate (L_t). Then the forward price contracted at time t and received at time T should equal

$$F_t^T = (S_t - L_t)e^{r*(T - t)}$$

Thus, if you have a stock that pays a dividend in half a year of d_{t1}, the equivalent loan value would be $L_t = d_{t1}e^{-rt1}$. Now if

$$F_t^T > (S_t - L_t)e^{r*(T - t)},$$

then an arbitrageur can make a riskless profit by borrowing S_t to buy the stock and sell a forward contract. Then she will have to pay back $S_te^{r*(T-t)}$ at time T. The present value of her income from the asset interest payments is L_t and the value at T equals $L_t*e^{r*(T-t)}$. Since she will receive F_t^T, her net profit will be $F_t^T - (S_t - L_t)e^{r*(T - t)}$.

To illustrate pricing futures with interest or dividend payments, take the example from Hull (2000), p. 39. Consider a 12 month forward contract on a 5 year bond currently selling for $900. The bond pays a $60 coupon payment at 6 months and at 12 months. The continuous compounding 6 month rate is 9% and the continuous compounding 1 year rate is 10%. First compute the discounted present value of your interest receipts as follows

$$L_t = 60e^{-0.09*0.5} + 60e^{-0.1*1} = \$111.65.$$

Second, we compute the forward price by substituting L_t by its value in the formula and we obtain

$$F_t^T = (900 - 111.65)e^{0.1*1} = \$871.26.$$

If the forward price were less than $871.26, the arbitrageur should short the bond and buy forward. If the forward price is greater than $871.26, the arbitrageur should short the forward contract and buy the bond.