

## Optimal Hedge Ratio

In hedging, you can hedge your whole portfolio or some portion of it. The hedge ratio is the size of the futures contract relative to the cash transaction. In a previous example involving a trader with oil en route from the Gulf, the hedge ratio was one, since she sold a futures contract representing each barrel of oil. When using turnips to hedge, the ratio was still 1, since the value of the crude was hedged with an equal value of turnips. If the hedger had sold  $\frac{1}{2}$  barrel for each barrel in transit, the hedge ratio would have been  $\frac{1}{2}$ . It turns out that it is not always optimal to hedge your whole product and Hull (2000) works out what the optimal hedge ratio is to minimize risk. In his exposition, he defines

$\Delta S = S_T - S_t$  = the change in the spot price over the life of the contract,

$\Delta F = F_T - F_t$  = the change in the futures price over the life of the contract,

$\sigma_S$  = the standard deviation of  $\Delta S$ ,

$\sigma_F$  = the standard deviation of  $\Delta F$ ,

$\rho = \sigma_{SF}/(\sigma_S\sigma_F)$  = the correlation coefficient between  $\Delta S$  and  $\Delta F$ , which is the covariance of  $\Delta S$  and  $\Delta F$  divided by the standard deviations of  $\Delta S$  and  $\Delta F$ , and

$h$  = the hedge ratio.

If the hedger owns the products and sells the future, his portfolio value is  $(S - hF)$ . The change in value of the portfolio is

$$\Delta S - h\Delta F.$$

Alternatively, if the hedger buys the future and is short the product, his portfolio value is  $hF - S$ . The change in value of the portfolio is

$$h\Delta F - \Delta S.$$

The variance ( $\sigma^2$ ) for the above two portfolios is

$$\sigma^2 = \sigma_S^2 + h^2\sigma_F^2 - 2h\sigma_{SF} = \sigma_S^2 + h^2\sigma_F^2 - 2h\rho\sigma_S\sigma_F.$$

To find the optimal hedge ratio, which minimizes risk or variance, minimize the above expression with respect to  $h$ .

$$\partial\sigma^2/\partial h = 2h\sigma_F^2 - 2\rho\sigma_S\sigma_F = 0.$$

Checking second order conditions

$$\partial^2\sigma^2/\partial h^2 = 2\sigma_F^2 > 0.$$

So  $h$  is a minimum. Solving for  $h$ , we get

$$h = \rho\sigma_S/\sigma_F.$$

Now if S and F are for the same products it is likely that  $\sigma_S$  and  $\sigma_F$  are close to the same value and  $\rho$  is close to 1. Then the optimal hedge ratio is near 1. Where this becomes more interesting is where you are hedging one product with a different products future contract. For example, you might use Henry Hub gas to hedge for gas at Waha or some other hub. Then  $\sigma_S$  and  $\sigma_F$  may not be close to the same and  $\rho$  may not be close to 1. The closer  $\rho$  is to one, and the larger is the variance of the product you are hedging, the more you hedge. The larger is the variance of the product used to hedge the lower the hedge ratio. It is even possible that h would be greater than 1.